Mathematics 172 Homework

Let us get back to the logistic equation equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where r is the intrinsic growth rate and K is carrying capacity. We know do some variants on this equation.

1. Let $P(t)$ be the number of grams of algae in an aquarium t months after it is set up. Originally the algae grows logistically with $r = .15$ and $K = 15$.

(a) What is the differential equation satisfied by P?

Answer: $\dfrac{dP}{dt} = .15P\left(1 - \dfrac{P}{15}\right)$.

A some point algae eating snails are introduced to the aquarium. They eat the algae are a rate of 10% of the amount present.

(b) What is the new differential equation satisfied by P?

Answer: $\dfrac{dP}{dt} = .15P\left(1 - \dfrac{P}{15}\right) - .1P$

(c) What is the stable population size of the algae after the snails are introduced?

Answer: $P = 5$.

(d) If the snails eat 20% of the amount of algae present, what is the new rate equation and stable population size?

Answer: $\dfrac{dP}{dt} = .15P\left(1 - \dfrac{P}{15}\right) - .2P$ and $P = 0$ (that is the algae dies out).

2. With the same logistic rate equation for the amount of algae in the aquarium, that is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right).$$

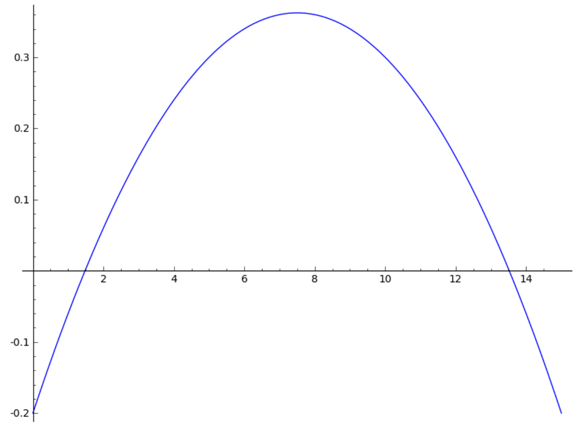
This time a filter is put in that removes the algae at a continuous rate of .2 grams/month.

(a) What is the new rate equation for P?

Answer: $\dfrac{dP}{dt} = .15P\left(1 - \dfrac{P}{15}\right) - .2$

(b) What are the equilibrium points of this equations?

Answer: We need to solve $.15P\left(1 - \dfrac{P}{15}\right) - .2 = 0$. This is not easy to do by hand, so we use the calculator. Use the Y= button and enter \`Y1 = .15*X*(1-X/15)-.2` Use the WINDOW button and `Xmin=0` and `Xmax=15`. Then do a `ZoomFit` to plot the function. You should get something that looks like:



Now use 2ND CALC and then the zero function to find that the two points where the graph crosses the x axis are $P = 1.479$ and $P = 13.521$. These are the equilibrium points.

(c) Which of these two points is stable? *Answer:* $P = 13.521$