Mathematics 172 Homework

1. Here are problems for practicing derivatives.

(a) $y = 4e^{2t}$.

Answer: $\dfrac{dy}{dt} = 2 \cdot 4e^{2t} = 8e^2$.

(b) $N(t) = 5e^{-2.2t}$.

Answer: $N'(t) = -2.2 \cdot 5e^{-2.2t} = -11e^{-2.2t}$.

(c) $P = 375e^{.0142t}$.

Answer: $\dfrac{dP}{dt} = .0142(375)e^{.0142t} = 5.325e^{.0142t}$.

2. Here are some derivative problems that have constants in them.

(a) $P = P_0e^{2t}$, with P_0 a constant.

Answer: $P' = 2P_0e^{2t} = 2P$.

(b) $N = N_0e^{-.4t}$, with N_0 a constant.

Answer: $\dfrac{dN}{dt} = -.4N_0e^{-.4t} = -.4N$.

(c) $A = A_0e^{.067t}$.

Answer: $\dfrac{dA}{dt} = .067A_0e^{.067t} = .067A$.

(d) $N = N_0e^{rt}$.

Answer: $\frac{dN}{dt} = rN_0e^{rt} = rN$.

At this point you have likely seen a pattern. That is if $y = y_0e^{rt}$, then $y' = ry$. Or in different notation $\dfrac{dt}{dt} = ry$. This has a converse.

Basic fact about exponentials. If $y = y(t)$ is a function of t, then

$$\frac{dy}{dt} = ry \qquad \text{implies} \qquad y = y_0e^{rt}.$$

3. Here are some problems to practice using this fact.

(a) If $\dfrac{dy}{dt} = 5y$ and $y(0) = 9$, then find $y(t)$.

Answer: Here $y_0 = 9$ thus $y(t) = 9e^{5t}$.

(b) If $\dfrac{dP}{dt} = .15P$, and $P(0) = 500$, find $P(t)$.

Answer: $P(t) = 500e^{.15t}$.

(c) If $P'(t) = -.25P$ and $P(0) = 100$, find $P(t)$.

Answer: $P(t) = 100e^{-.25t}$.

(d) If $A'(t) = rA(t)$, $A(0) = 10$, $A(4) = 25$, and r is a constant, then find a formula for $A(t)$ and the value $A(25)$.

Answer: By the Basic Fact, we have that $A(t) = 10e^{rt}$. Then

$$A(4) = 10e^{5r} = 25$$

and so as we have done before this leads to

$$r = \frac{25/10}{4} = .22907$$

Thus

$$A(t) = 10e^{.22907t}$$

and

$$A(25) = 10e^{.22907(25)} = 3069.69$$

(e) If $P'(t) = rP(t)$, $P(0) = 78$, $P(2) = 83$ and r is constant. Then find a formula for $P(t)$, the value of $P(10)$ and how long it takes for $P(t)$ to reach 1000.

Answer: $P(t) = 78e^{.03107t}$, $P(10) = 106.4$, and the time it takes to get to 1000 is $t = 32.19$