Mathematics 300 Test 1 Name: _____

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (5 points) Let $\mathbb{N}$ be the natural numbers. Let $A = \{x \in \mathbb{N} : 3 < x \leq 7\}$.

(a) Write A as a list of elements between brackets. (That is of the form $\{a, b, c, \ldots\}$).

$A =$ _____

(b) What is $|A|$? $|A| =$ _____

2. (15 points) (a) Define n is an ***even integer***.

(b) Define n is an ***odd integer***.

(c) Define n is ***divisible by*** 3.

Let
$A = \{n \in \mathbb{N} : 1 \leq n \leq 18 \text{ and } n \text{ is even.}\}$
$B = \{n \in \mathbb{N} : 1 \leq n \leq 18 \text{ and } n \text{ is even.}\}$
$C = \{n \in \mathbb{N} : 1 \leq n \leq 18 \text{ and } n \text{ divisible by } 3.\}$

Write the following as a a list of elements between brackets.

$A =$ _____

$B =$ _____

$C =$ _____

$A \cap B =$ _____

$A \cap C =$ _____

$A - C =$ _____

3. (10 points) (a) For a set, A, define the ***power set***, $\mathcal{P}(A)$ of A.

(b) Let $A = \{1, 2, 5\}$ then what is $\{X \in \mathcal{P}(A) : |X| = 2\}$

$\{X \in \mathcal{P}(A) : |X| = 2\} =$ _____

4. (10 points) (a) Draw the Venn diagram for $\overline{A} \cap (B \cup C)$.

(b) Draw the Venn diagram for $(A \cup C) - B$.

(c) Does $\overline{A} \cap (B \cup C) = (A \cup C) - B$ hold for all sets A, B, C? Explain your answer. *Hint:* Use your answers to parts (a) and (b).

5. (5 points) If $[a, b]$ is the set of real numbers, x, with $a \leq x \leq b$ find the union

$$\bigcup_{n=0}^{5} [n, n+1]$$

The union is _____

6. (5 points) Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y \geq 0\}$. Shade the region $A \cap B$.

7. (15 points) Make truth tables for the following:

(a) $P \wedge Q$

(b) $P \implies Q$

(c) $P \iff \sim Q$

(d) $(P \wedge \sim Q) \vee ((\sim P) \wedge Q)$

(e) Does the logical equivalence $P \iff \sim Q = (P \wedge \sim Q) \vee ((\sim P) \wedge Q)$ hold? Explain your answer. *Hint:* Use your answers to (c) and (d).

8. (10 points) Write out the negations of the following statements.

(a) He will study for the test, but will fail it.

(b) Every one in this call will get an A on this test.

9. (5 points) State the ***division algorithm***.

10. (10 points) (a) For integers a, b, n with $n \geq 1$ define $a \equiv b \mod n$.

(b) Prove that if $a \equiv b \mod n$, then $3a \equiv 3b \mod 3n$.

11. (10 points) Give a contrapositive proof that if q^2 is even, then q is even.