Mathematics 300

Quiz 18 Name: Key

You must show your work to get full credit.

We have shown the following in class:

Proposition. *If a is an integer and a^2 is odd, then a is odd.*

You can use this proposition in doing the following:

1. Show that if a and b are integers, then $a^2 - 4b - 3 \neq 0$.

<u>Proof</u> Towards a contradiction assume that there exist integers a and b with

$$(*) \qquad a^2 - 4b - 3 = 0$$

Then
$$a^2 = 4b+3$$
$$= 2(2b+1)+1$$

and $2b+1 \in \mathbb{Z}$ so a^2 is odd.
By the proposition this implies a is odd.
Thus $a = 2k+1$ for some integer. Use $a = 2k+1$ in equation$(*)$ to get

$$(2k+1)^2 - 4b - 3 = 0$$
$$4k^2 + 4k + 1 - 4b - 3 = 0$$
$$4k^2 + 4k - 4b = 2$$
$$4(k^2 + k - b) = 2$$
$$2(k^2 + k - b) = 1$$

and $k^2 + k - b \in \mathbb{Z}$ which implies 1 is even. But 1 is not even, a contradiction.

<u>done</u>