

At $x = u$, $\boxed{}$ has the indeterminate form $\boxed{}$ if $\lim_{x \rightarrow u} f(x) = \boxed{}$ and $\lim_{x \rightarrow u} g(x) = \boxed{}$

(1)	$\frac{f(x)}{g(x)}$	$\frac{0}{0}$	0	0
(2)	$\frac{f(x)}{g(x)}$	$\frac{\infty}{\infty}$	∞	∞
(3)	$f(x) \cdot g(x)$	$0 \cdot \infty$	0	∞
(4)	$f(x) - g(x)$	$\infty - \infty$	∞	∞
(5)	$[f(x)]^{g(x)}$	0^0	0	0
(6)	$[f(x)]^{g(x)}$	∞^0	∞	0
(7)	$[f(x)]^{g(x)}$	1^∞	1	∞

HERE: u stands for any of the symbols a , a^- , a^+ , $-\infty$, $+\infty$.

L'Hôpital's Rule

(1) and (2)

If:

- $\frac{f(x)}{g(x)}$ has the indeterminate form $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$ at u

and

- $\lim_{x \rightarrow u} \frac{f'(x)}{g'(x)}$ exists (i.e. this limit is a finite number or $-\infty$ or ∞)

then

$$\lim_{x \rightarrow u} \frac{f(x)}{g(x)} = \lim_{x \rightarrow u} \frac{f'(x)}{g'(x)} .$$

(3)

If $f(x) \cdot g(x)$ has the indeterminate form $\boxed{0 \cdot \infty}$ at u , then rewrite:

$$f(x) \cdot g(x) = \frac{f(x)}{1/g(x)} , \text{ which has the indeterminate form } \boxed{\frac{0}{0}} \text{ at } u$$

or

$$f(x) \cdot g(x) = \frac{g(x)}{1/f(x)} , \text{ which has the indeterminate form } \boxed{\frac{\infty}{\infty}} \text{ at } u$$

and then apply L'Hôpital's Rule.

(4)

If $f(x) - g(x)$ has the indeterminate form $\boxed{\infty - \infty}$ at u ,

then use algebraic manipulation to convert $f(x) - g(x)$

into a form of the type $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$

and then apply L'Hôpital's Rule.

(5)

If $[f(x)]^{g(x)}$ has the indeterminate form $\boxed{0^0}$ at u , then follow these steps:

Let

$$y = [f(x)]^{g(x)} .$$

So

$$\ln y = \ln \left([f(x)]^{g(x)} \right) .$$

Next, simplify

$$\ln y = [g(x)] \cdot \ln [f(x)] .$$

Note that $\ln y = [g(x)] \cdot \ln [f(x)]$ has the indeterminate form $\boxed{0 \cdot -\infty}$ at u .

Using an appropriate above method (i.e. (3)), evaluate

$$\lim_{x \rightarrow u} \ln y \equiv L .$$

Conclude

$$\lim_{x \rightarrow u} \ln [f(x)]^{g(x)} = L \quad \implies \quad \lim_{x \rightarrow u} [f(x)]^{g(x)} = e^L .$$

(6) and (7)

If $[f(x)]^{g(x)}$ has the indeterminate form $\boxed{\infty^0}$ or $\boxed{1^\infty}$, then proceed similarly as in (5) .

Note that $\ln y = [g(x)] \cdot \ln [f(x)]$ will have the indeterminate form

$$(6) \quad \boxed{0 \cdot \infty} \text{ at } u$$

$$(7) \quad \boxed{\infty \cdot 0} \text{ at } u .$$