Mathematics 242 Homework.

Problem 1. For the equation

$$mx'' + kx = F_0 \sin(\omega t)$$

we divide by m to get

$$x'' + \omega_0^2 x = \frac{F_0}{m} \sin(\omega t)$$

where we have set

$$\omega_0 = \sqrt{\frac{k}{m}}.$$

(a) Derive the general solution to this equation in the case where $\omega_0^2 \neq \omega^2$.
(b) Derive the general solution to this equation in in the case $\omega_0^2 = \omega^2$. □

Problem 2. (a) Find the general solution to

$$9x'' + 4x = \sin(2t/3).$$

(which it the equation for a mass of 9 grams on spring with spring constant 4).

(b) show that all of these solutions become unbounded. *Hint:* To show the solutions are unbounded it is OK with me just to graph the particular solution to see it is unbounded and note that the general solution to the homogeneous problem is bounded. □

Problem 3. Find the solution to initial value problem

$$4x'' + 4x' + 5x = 0, \qquad x(0) = 3, \quad x'(0) = 0.$$

Is this spring system over or under damped?

Problem 4. Let us add a forcing function to equation of the last problem. Find the solution to

$$4x'' + 4x' + 5x = .1\cos(t/2), \qquad x(0) = 3, \quad x'(0) = 0.$$

Problem 5. The characteristic equation to the equation

$$y^{(4)} - 3y''' + 7y'' + 21y' - 26y = 0$$

is

(1) $$r^4 = 3r^3 + 7r^2 + 21r - 26 = 0$$

which has roots

$$r = 1, \quad -2, \quad 2 + 3i, \quad 2 - 3i.$$

(a) What is the general solution to the homogeneous equation (1)?
(b) What is the general solution to the inhomogeneous equation

$$y^{(4)} - 3y''' + 7y'' + 21y' - 26y = 5e^{3x}.$$

□

Problem 6. The homogeneous equation
$$y^{(4)} - 3y''' + 6y'' + 28y' - 24y = 0$$
has characteristic polynomial

(2) $$P(r) = r^4 - 3r^3 - 6r^2 + 28r - 24$$

which factors as
$$P(r) = (r - 2)^3(x + 3).$$

(a) What is the general solution to homogeneous equation (2)?

(b) What is the general solution to
$$y^{(4)} - 3y''' + 6y'' + 28y' - 24y = 12x - 24?$$

Problem 7. Find a particular solution to
$$y''' + 3y'' + 2y' + 4y = 3\cos(2x) + 4\sin(2x).$$