Mathematics 172 Homework, August 24, 2023.

We can going to start out a little bit of dimensional analysis and apply it to scaling. Most numbers in that come up it the sciences have some sort of unit (also called a dimension in some contexts) associated with them. For example if you are talking about someones height, you say she is 5.5 ft, not just 5.5. Likewise if you are asking bout the weight of a dog, and just get the answer 10, there is quite a difference between 10 lb and 10 kg.

There are two rules about doing algebra with quantities with units.

- You can only add or subtract terms with the same units.
- In doing multiplication or division of terms you multiply or divide the units.

Some examples. It $a = 3\,\text{hr}$ and $b = 6\,\text{hr}$, then

$$a + b = 9\,\text{hr}, \qquad a - b = -3\,\text{hr}.$$

If $x = 4$ lb and $y = 7\,\text{sec}$ then

$$x + y = \text{not defined}$$

as adding time and weights does not make sense.

On the other hand it $D = 30\,\text{miles}$ and $t = 2\,\text{hr}$ then

$$Dt = 60\,\text{miles hr}$$

and

$$\frac{D}{t} = 15(\,\text{miles}/\,\text{hr}).$$

1. It $t_1 = .5se$, $t_2 = 3\,\text{sec}$ and $D = 4.5\,\text{m}$ (here m is meters) what are the units of $\dfrac{D}{t_1 t_2}$. *Solution:*

$$\frac{D}{t_1 t_2} = \frac{4.5\,\text{m}}{(.5m)(3\,\text{m})} = 3(\,\text{m}/\,\text{sec}^2).$$

Here are some conversions between some common units. In the following L stands for liters.

$$\begin{array}{cc} 1\,\text{cm} = 0.393701\,\text{in} & 1\,\text{in} = 2.54\,\text{cm} \\ 1\,\text{miles} = 1.60934\,\text{km} & 1\,\text{km} = 0.621371\,\text{miles} \\ 1\,\text{gal} = 3.78541\,\text{L} & 1\,\text{L} = 0.264172\,\text{gal} \end{array}$$

I am assuming your the basic conversions of the metric system, i.e. $1\,\text{cm} = .1\,\text{kg}$ etc. And that there are $1\,\text{min} = 60\,\text{sec}$ and the like. We can use these to do more complicated conversion.

Example 1. Chance $v = 45\,\text{miles}/\,\text{hr}$ to $\,\text{m}/\,\text{hr}$.

Solution. Just plug in the conversion between miles and meters and then change to meters.

$$\begin{aligned} v &= 45\frac{\text{miles}}{\text{hr}} \\ &= 45\frac{1.60934\,\text{kg}}{\text{hr}} \\ &= 45\frac{1.60934(1{,}000\,\text{m})}{\text{hr}} \\ &= 72420.3(\,\text{m}/\,\text{hr}) \end{aligned}$$

Example 2. Change $r = 30\,\text{gal}/\,\text{min}$ to $\text{L}/\,\text{sec}$.

Solution. The same basic idea as the previous example.

$$\begin{aligned} r &= 30\frac{\text{gal}}{\text{min}} \\ &= 10\frac{3.78541\,\text{L}}{60\,\text{sec}} \\ &= 0.630901\,\text{L}/\,\text{sec}. \end{aligned}$$

Here are a few for you try.

2. Change $100\,\text{kg}/\,\text{hr}$ to miles/hour.

3. Change $32\,\text{ft}/\,\text{sec}^2$ to $\text{m}/\,\text{sec}^2$.

4. Change $.23\,\text{lb}/\,\text{in}^3$ to $\text{kg}/\,\text{cm}^3$.