Mathematics 172 Homework, August 29, 2023.

Let $\mathcal{A}$ be some object or organism, say a dog. Let $\mathcal{B}$ be a copy of $\mathcal{A}$ that is ***magnified*** (or ***scaled***) by a factor of λ. So if $\lambda = 2$ we have a copy that is exactly twice as large as $\mathcal{A}$. If $\lambda = .5$ then $\mathcal{B}$ is one half as large as $\mathcal{A}$.

The basic idea of scaling is to understand how various measurements change under scaling:

Measurements with units of length (say cm)	Scale by λ
Measurements with units of area (say cm^2)	Scale by λ^2
Measurements with units of volume (say cm^3)	Scale by λ^3
Measurements with units of mass or weight (say kg)	Scale by λ^3

1. A tank contains 75 gallons of water. How much does a tank that is scaled by a factor of 3 (that is three times as large) contain?

Solution: Here we have $\lambda = 3$ and we are measuring volume so the volume scales by λ^3 thus the volume of the larger tank is

$$(3)^3 75 = 2025 \, \text{gallons}.$$

2. The surface area of the tank in Problem 1 is 100 ft^2. What is the surface area of the tank that is scaled by a factor of $\lambda = .5$ (that is half as large as the orginial tank).

Solution: This scales by λ^2 so the new surface area is $(.5)^2 100 = 25 ft^2$,

3. If we have two fish with the same shape, by one is 30cmlong and the second is 45cmlong.

(a) What is the scaling factor, λ, to scale the smaller fish up to the size of the larger fish.

Solution: We need to solve $\lambda 30 \text{ cm} = 45 \text{ cm}$. This gives $\lambda = 45/30 = 1.5$.

(b) If the smaller fish weighs 0.3 kg, then what is the weight of the larger fish?

Solution: Weight scales by λ^3 so the weight of the larger fish is $(1.5)^3(.3) = 1.0125\text{kg}$.