Mathematics 172 Homework, September 7, 2023.

1. In this problem we use scaling to see that there is a lower limit on the size of dolphin that can live in cold water. We assume the dolphins are living in water that is 2°C. Also assume

- The body temperature of a dolphin is 36°C and that at this body temperature the dolphin loses energy through its skin at a rate of 35(cal/m^2)/hr.
- The maximum rate a dolphin can produce energy is 1.45(cal/kg)/hr.
- A dolphin that is 2 meters long weights 65kg and has a surface area of 1.8m^2.

We will use this information to find the length of the smallest dolphin that can maintain its body temperature in 2°C water.

To do this we consider a dolphin which is a scaled, by a factor of λ, of the 2m dolphin described above.

(a) What is the weight, W_λ, of the scaled dolphin?
(b) What is the surface area, A_λ, of the scaled dolphin?
(c) What is total energy loss, E_λ, per hour of the scaled dolphin?
(d) What is the maximum total energy that the scaled dolphin can produce?
(e) Write an equation which says the maximum total energy produced by the scaled dolphin equals the total energy loss. At this size (i.e. choice of λ) the dolphin is using all its energy just to keep its temperature at 36°C.
(f) Solve the equation for λ.
(g) What is the minimal length of a dolphin that can live in 2°C water?

Answers on next page.

(a) Weight scales like λ so

$$W_\lambda = 65\lambda^3\text{kg}$$

(b) Surface area scales by λ so

$$A_\lambda = 1.8\lambda^2\text{m}^2$$

(c) The total loss of energy is the rate of loss per m^2 times the total number of m^2, that is times the surface area.

$$\begin{aligned}\text{total loss}_\lambda &= \big(35(\text{cal}/\text{m}^2)/\text{hr}\big)\big(1.8\lambda^2\text{m}^2\big)\\ &= 63\lambda^2\text{cal}/\text{hr}.\end{aligned}$$

(d) The maximum energy that can be produced by the dolphin is the maximum per kg times the weight.

$$\begin{aligned}\text{max energy}_\lambda &= \big(1.45(\text{cal}/\text{kg})/\text{hr}\big)\big(65\lambda^3\text{kg}\big)\\ &= 94.25\lambda^3\text{cal}/\text{kg}\end{aligned}$$

(e) The equation is

$$\text{total loss}_\lambda = \text{max energy}_\lambda.$$

That is

$$63\lambda^2\text{cal}/\text{hr} = 94.25\lambda^3\text{cal}/\text{kg}.$$

Canceling some of the λs and all of the units gives

$$63 = 94.25\lambda.$$

(f) Solving for λ gives

$$\lambda = \frac{63}{94.25} = .668$$

(g) The minimal length of a dolphin which will not lose its body heat is that $\lambda\times 2$ (we have been scaling from a lemgth anaimal). So the mimimum length is $.668 \times 2 = 1.336$m.