Mathematics 300 Homework, August 21, 2024.

You should read over the material in the text about closure properties of the various number systems. This on pages 11–12 of the text.

Definition 1. Let n be an integer.

- n is ***even*** if and only if there is an integer q such that $n = 2q$.
- n is ***odd*** if and only if $n = 2q + 1$ for some integer q.

This 0 is even as $0 = 2 \cdot 0$, that is $0 = 2q$ with $q = 0$.

Problem 1. (a) Show that -11 is odd by finding a q so that $-11 = 2q+1$.
(b) Show that for any integer k the integer $2k - 1$ is odd (that is find q so that $2k - 1 = 2q + 1$).
(c) Show that if x odd and y is even that $x + y$ is odd. □

Problem 2. Prove or give a counter example: If a is odd, then $a^2 + a$ is even. □

Problem 3. Prove or give a counterexample: If a is even, then $a^2 + a$ is even. □

Problem 4. Prove or give a counterexample: If x is an integer, then $x^2 + 4x + 1$ is odd. □

Problem 5. Prove or give a counterexample: If x and y are integers, then $2xy^2 - 11$ is odd. □

Problem 6. Prove: If x is an odd integer, then x^3 is odd.