**Mathematics 122**

**Quiz #23** Name: Key

***You must show your work to get full credit.***

Let $a > 0$ be a constant and set

$$f(x) = ax^2 - x^3 \qquad \text{where} \qquad 0 \le x \le a.$$

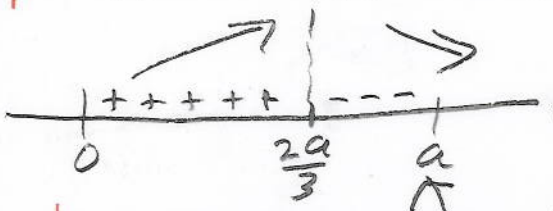
1 pt (1) Find the critical point(s) of $f(x)$ in the interval $0 < x < a$.

$f'(x) = 2ax - 3x^2$
$= x(2a - 3x) = 0$
so $x = 0$ or $(2a - 3x) = 0$
i.e. $x = 0$ or $x = \frac{2a}{3}$

Critical Point(s) = $\frac{2a}{3}$

But 0 is not between 0 and $a$ so only $x = \frac{2a}{3}$ counts.

1 pt (2) Where is $f(x)$ increasing on the interval $0 \le x \le a$?




$0 \le x \le \frac{2}{3}a$ or $0 < x < \frac{2}{3}a$

1 pt (3) What is the maximizer of $f(x)$ on $0 \le x \le a$?

From diagram $x = \frac{2a}{3}$ is maximizer

$x = \frac{2a}{3}$

1 pt (4) What are the maximum value of $f(x)$ on $0 \le x \le a$?

maximum $= f(\frac{2a}{3})$
$= a(\frac{2a}{3})^2 - (\frac{2a}{3})^3$
$= \frac{4a^3}{9} - \frac{8a^3}{27}$
$= \frac{12a^3 - 8a^3}{27}$
$= \frac{4a^3}{27}$

$\frac{4a^3}{27}$ (or $a(\frac{2a}{3})^2 - (\frac{2a}{3})^3$)

1 pt. (5) Give a rough graph of $y = f(x)$ for $0 \le x \le a$ labeling the maximum point.