Mathematics 300

Quiz 12 Name: Key

You must show your work to get full credit.

1. A proof of the statement: "If $n+1$ is odd, then n^2+1 is even" starts with

Proof. Assume if $n+1$ is odd, then n^2+1 is even. ...

What is wrong with this?

The proof is assuming what is to be proven.

Proposition. *If $3n^2+1$ is even, then n is odd.*

2. What is the contrapositive of the proposition?

If n is even, then $3n^2+1$ is odd.

3. Prove the contrapositive.

Assume n is even. Then $n = 2q$ for some integer q. Then (substitution)

$$\begin{aligned} 3n^2+1 &= 3(2q)^2+1 \\ &= 12q+1 \\ &= 2(6q)+1 \\ &= 2k+1 \end{aligned}$$

where $k = 2q$ is an integer. (At this point we can stop saying "by closure properties" in the cases where it is obvious.)

Thus $3n^2+1 = 2k+1$ is odd by definition.