Mathematics 300

Quiz 7 Name: Key

You must show your work to get full credit.

1. For integers a and b define a ***divides*** b.

$a \neq 0$ and for some integer q, $b = qa$

2. Prove or give a counterexample. For all integers n, $2n^2 + n + 2$ is even.

False. The easiest counterexample is $n = 1$
Then $2n^2 + n + 2 = 2 + 1 + 2 = 5$ is not even

3. Prove or give a counterexample. If a divides x and b divides y, then ab divides $xy^2 + xy$.

Proof Assume $a | x$ and $b | y$. Then there are integers q_1 and q_2 with

$$x = q_1 a, \quad y = q_2 b.$$

so

$$xy^2 + xy = (q_1 a)(q_2 b)^2 + (q_1 a)(q_2 b)$$
$$= q_1 a q_2^2 b^2 + q_1 q_2 a b$$
$$= (q_1 q_2^2 b + q_1 q_2) ab$$
$$= q\, ab$$

where $q = q_1 q_2^2 b + q_1 q_2$ is an integer by closure properties of integers.

Thus $ab | (xy^2 + xy)$. done