Mathematics 300 Homework.

This homework is based on Section 3.1, Pages 82–90. The following definition will be on the quiz on Monday.

Definition 1. If a and b are integers with $a \neq 0$, then a ***divides*** b if and only if there is an integer q such that $b = qa$.

We use the notation $a \mid b$ for "a divides b". When this holds we say that a is a ***divisor*** or a factor of b and that b is a ***multiple*** of a. If a does not divide b, then we write $a \nmid b$.

Here are examples of using this definition in proofs.

Proposition 2. *If a divides b and a divides c, then a divides difference $b-c$.*

Proof. Assume that $a \mid b$ and $a \mid c$. Then by definition there are integers q_1 and q_2 such that

$$\begin{aligned} b &= q_1 a \\ c &= q_2 a. \end{aligned}$$

Then

$$b - c = q_1 a - q_2 a = (q_1 - q_2)a = q' a$$

where $q' = q_1 - q_2$ is an integer by the closure properties of the integers. Therefore $a \mid (b - c)$. □

Here the symbol □ is used to indicate the end of the proof.

Here is anther example:

Proposition 3. *If $n \mid x$ and $n \mid y$, then $n^2 \mid (2x^2 + 4xy)$.*

Proof. We are give that $n \mid x$ and $n \mid y$ and therefore there are integers k and ℓ such that

$$\begin{aligned} x &= kn \\ y &= \ell n. \end{aligned}$$

Then

$$\begin{aligned} 2x^2 + 4xy &= 2(kn)^2 + 4(kn)(\ell n) \\ &= 2k^2 n^2 + 4k\ell n^n \\ &= (2k^2 + 4k\ell)n^2 \\ &= qn^2 \end{aligned}$$

where $q = 2k^2 + 4\ell k$ is an integer by the Closure properties of the integers. Thus $n^2 \mid (2x^2 + 4xy)$. □

Assigned problems: On Page 96 of the text do problems 1 (part (a) is done above), 2ad, 3bdefg.