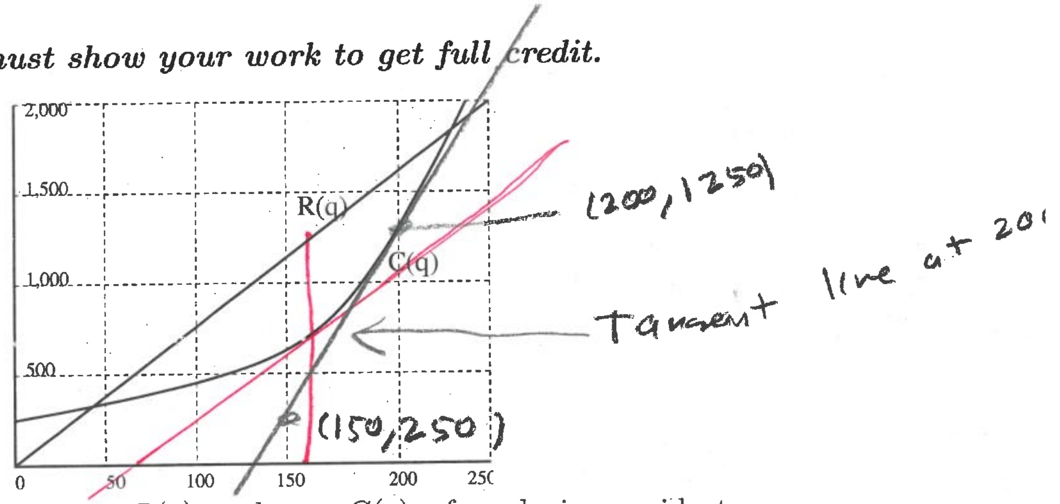
Mathematics 122

Quiz 11

Name: Key

You must show your work to get full credit.



1. The graph above gives the revenue, $R(q)$, and cost, $C(q)$, of producing q widgets.

(a) What is the revenue of selling a single widget. Revenue is \$8/widget

The average revenue/widget $= \frac{R(250)-R(0)}{250-0} = \frac{2000-0}{250} = 8$

As slope is constant this is the revenue of each widget

(b) Estimate the cost of producing the 201st widget. HINT: This is the same as estimating the marginal revenue $MC(200) = C'(200) =$ slope of tangent line when $q = 200$

$C'(200) \approx$ \$20/widget

This is slope of tangent line

I choose the points (200,1250) and (150,250) on the tangent line to get a slope of

$$\frac{\Delta C}{\Delta q} = \frac{1250-250}{200-150} = \frac{1000}{50} = 20$$

(c) If 200 widgets are being produced, should production be increased or decreased and why?

It should be decreased as the cost of producing another widget is greater than the revenue it produces

(d) Estimate the number of widgets that should be sold to maximize the profit. $q \approx$ 160 widgets

This is where $R'(q) = C'(q)$, i.e. where tangent line to $y = C(q)$ is parallel to $y = R(q)$ (shown in red).

This seems to be $q \approx 160$

2. Compute the following derivatives: (a) $y = 4x^5 - 9x^2 + 7$ $\quad \frac{dy}{dx} = 20x^4 - 18x$

(b) $A(r) = \frac{4}{r^3} = 4x^{-3}$ $\quad A'(r) = -12x^{-4} = \frac{-12}{x^4}$

(c) $g(t) = 6\sqrt{t} = 6t^{\frac{1}{2}}$ $\quad g'(t) = 3t^{-\frac{1}{2}} = \frac{3}{\sqrt{t}}$